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AN ANALYSIS OF ADJUSTMENT PROCEDURES

FOR GRENADE LAUNCHERS

by

Donald R. Barr

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ABSTRACT

A situation is considered in which a grenade launcher fires at a target, observes the projectile impact, adjusts the point of aim, fires a second round, adjusts, fires, and so on. A model is developed, based on the assumptions of normal ballistics, perfect observation of impacts, adjustments without error, and a unimodal target destruction function. The problem is to determine optimal adjustments, in order to maximize the probability of target destruction within a given number of rounds. It is shown that seemingly different adjustment procedures are equivalent, if viewed in appropriate coordinate systems. Previous results concerning sequential adjustments which are constrained to be linear functions of observed impact points are extended to the class of translation invariant procedures. Properties of the optimal sequential adjustment procedure, including some related to stochastic approximation, are reviewed. The effects of errors in judging impact positions are discussed.

Prepared by:

1. Introduction.

Consider the problem of adjusting grenade launcher fire on a target, using a "shoot-adjust-shoot" strategy. After each round is fired, the grenadier observes the projectile impact, and estimates its distance from the target. He then attempts to adjust his point of aim, so as to place the mean of the ballistic distribution of impact points directly over the target. The problem we address is that of determining an optimal adjustment scheme.

In what follows, we assume the ballistic distribution of impacts of rounds fired at a given aimpoint is bivariate normal. We further assume that the target is a point target, and that the target destruction function (or lethality function) d is the same for all rounds fired. The value of $d(x,y)$ is defined to be the probability that the target is destroyed by a round impacting at the point (x,y) relative to the target. The difficulty of finding optimal adjustment procedures hinges initially upon how errors are made by the grenadier in estimating the position of each impact point, relative to the target. For the most part, we confine ourselves to the simplest case in which the grenadier is "perfect"--he delivers adjustments which are exactly the desired proportions of the observed miss distances. In the last section, some remarks are made concerning the probable effects of errors in this process.

The problem of how to optimally adjust fire on a stationary point target has been of considerable interest. Grubbs [8] gives an adjustment scheme which is "best" (i.e. minimum variance) and unbiased among linear functions of the impact points. (In the literature of statistical estimation, such a scheme is called "BLUE", for "best linear unbiased

estimator.") Nadler and Eilbott [10] derive the BLUE adjustment scheme and show it maximizes, among linear functions of the impact points, the probability of destroying the target with each round, for a fairly general class of destruction functions. This procedure is a special case of stochastic approximation schemes, about which a great deal, mostly of an asymptotic nature, has been written [5], [11], [12]. Other stochastic approximation approaches to the adjustment of fire have included cases in which the observer cannot precisely locate the impact points (see, for example, Barr and Piper [3]).

The main purpose of this paper is to demonstrate that, with appropriate coordinate systems, the procedures of Grubbs and of Nadler and Eilbott may be obtained from well known statistical theory concerning BLUE estimators. We also show (in Section 2) that these procedures are indeed stochastic approximation procedures, and (in Section 3) that they possess an optimality property stronger than BLUE. The present approach seems notationally and conceptually simpler than those used previously.

It is notationally convenient to restrict our consideration to a single dimension, say range, in what follows. Under the assumptions we make below (which are somewhat weaker than those of Nadler and Eilbott), the restriction to one dimension causes no loss of generality: identical optimal one-dimensional procedures carried out independently in each dimension gives an optimal two-dimensional procedure. (See Section 5 below.)

2. The Model.

Assumptions which permit analysis of a two-dimensional problem by separate analyses with each dimension are essentially those required for Anderson's [2] inequality. The following are sufficient:

i) statistical independence of the range and deviation components of the impact points, each component having a symmetric unimodal continuous distribution,

ii) no grenadier errors are made, and

iii) the destruction function d is symmetric about the impact point (x,y) and $\{(x,y): d(x,y) \geq \alpha\}$ is convex for all $\alpha \geq 0$.

In particular, a circular normal ballistic distribution with a destruction function of the "circular normal" or "circular cookie-cutter" types will satisfy condition (iii). It is well known that in actual gunnery problems, the ballistic distribution is such that the variance in range is not the same as that in deviation. In most gunnery problems, these variances depend on many factors, but principally upon the grenade launcher-target range. The assumption that the impact points fall in a circular normal pattern about the aimpoint is equivalent to agreeing to measure range and deviation in the corresponding standard deviation units, and in addition, assuming independence between the range and deviation components of the impact points.

Suppose, then, that $X_1^1, X_2^1, X_3^1, \ldots$ are independent normal random variables with unknown mean μ and known variance σ^2 (we may take $\sigma^2 = 1$ without loss of generality*). These random variables represent the sequence of impact point ranges that would be obtained with a sequence of rounds fired without adjustment (i.e. with a fixed aim point). The problem is to determine a sequence of adjustments, $\xi_1, \xi_2, \ldots$, where ξ_n is the adjustment made between the $n^{\underline{th}}$ and $n+1^{\underline{st}}$ rounds, so that

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*As mentioned above, assuming $\sigma^2 = 1$ is equivalent to measuring distances in σ-units. In theory, σ^2 varies with the range being fired, and hence varies as adjustments in range are made. However, in practice, this variation in σ^2 is sufficiently small within a given firing mission that σ^2 may be assumed to be constant.

the probability of target destruction with each round is maximized. (As we remark below, this will maximize the probability of target destruction within any fixed number of rounds.)

In general, the $n^{\underline{th}}$ adjustment is a function of the first n observed impact ranges, $X_1, X_2, \ldots, X_n$. Each adjustment ξ has the effect of translating the coordinate system $-\xi$ units. For example, if an adjustment $\xi = -50$ (corresponding to the intention of dropping 50 meters in range) is made, the net effect is as if the coordinate system (i.e. target) were transformed 50 units away from the grenade launcher. We must therefore keep track of the adjustments made up to the $n^{\underline{th}}$ round, so as to be able to express the $n^{\underline{th}}$ impact point in coordinates which are meaningfully related to the previous impact points. Accordingly, let X_j^r denote the $j^{\underline{th}}$ impact range, expressed in the $r^{\underline{th}}$ coordinate system. Upon firing the first round, the grenadier observes X_1^1, and makes adjustment ξ_1. The first round can thus be given as $X_1^2 = X_1^1 + \xi_1$. Similarly, the second round X_2^2, fired after the adjustment ξ_1, can be given in the initial coordinate system as $X_2^1 = X_2^2 - \xi_1$, and so on for subsequent rounds, adjustments and coordinate systems.

Remark. It should be noted that the procedure we will determine does not require the grenadier to actually estimate the miss distances X_i^i. Rather, it will turn out that he should attempt to adjust by a fraction of the observed miss distance. Thus, the grenadier needs only to be able to duplicate some fraction of an observed distance in his aim adjustment. The present discussion involving estimates X_i^i of actual distance values are a theoretical convenience only.

If all observed impact ranges are transformed to the initial coordinate system, and if each adjustment is some linear function of the data available for that adjustment (i.e. $\xi_n(X_1^1, X_2^2, \ldots, X_n^n)$), then the ξ's are also linear functions of the "unadjusted" impact ranges $X_1^1, X_2^1, \ldots, X_n^1$. The problem then becomes that of finding the optimal linear function of a random sample of identical, independent normal $(\mu,1)$ random variables. Naturally, one expects the sample mean $\bar{X}_n^1 = \frac{1}{n} \sum_{i-1}^{n} X_i^1$, to provide an optimal adjustment $-\xi_n^1$, <u>from the initial aimpoint μ</u>, where we write "ξ_n^1," rather than "ξ_n" to indicate we mean adjustment from the initial aimpoint. In what follows, we call this the "X-bar procedure."

It is easy to establish that, in fact, $\xi_n^1 = -\bar{X}_n^1$ is the optimal linear adjustment. It is well known that $\bar{X}_n^1$ is a BLUE estimator for μ under the present conditions.* With the unimodal, symmetric target destruction function considered here it is clear that one may restrict his attention to linear adjustments which are unbiased estimators of $-\mu$, since otherwise the probability of target destruction on the next round could be made as high or higher by subtracting the bias. But, among the normally distributed aimpoints for the $n + 1^{st}$ round, which arise after applying unbiased adjustments ξ_n^1, the probability of target destruction is maximized by minimizing the variance of the aimpoints about the target. That is precisely what the BLUE estimator of $-\mu$ achieves, and it is the optimal scheme developed by the aforementioned authors.

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*For example, Larson [9], gives the following [Theorem 7.3.1]: X is a random variable with mean μ and variance σ^2. If $X_1, X_2, \ldots, X_n$ is a random sample of X, $\bar{X}$ is the best linear unbiased estimator of μ.

In order to see that the X-bar procedure $\{\xi_n^1\}$ is a stochastic approximation procedure, we shall express it in terms of the sequence of current coordinate systems, $\{\xi_j^j\}$. Note that

$$\xi_1^1 = -X_1^1 = -\bar{X}_1^1.$$

After making this adjustment and firing the second round, resulting in X_2^2, we have, in the second coordinate system, two observations: X_1^2 and X_2^2. Now

$$\begin{aligned}\xi_2^2 &= \xi_2^1 - \xi_1^1 \\ &= -\frac{1}{2}(X_2^1+X_1^1) - \xi_1^1 \\ &= -\frac{1}{2}(X_2^2-\xi_1^1-\xi_1^1) - \xi_1^1 \\ &= -\frac{1}{2}X_2^2.\end{aligned}$$

In general, $\xi_n^n = \xi_n^1 = \xi_{n-1}^1$

$$\begin{aligned}&= -\frac{1}{n}(X_n^1+(n-1)\xi_{n-1}^1) - \xi_{n-1}^1 \\ &= -\frac{1}{n}X_n^n.\end{aligned}$$

Thus, the optimal "X-bar" procedure is identical to the "$\frac{1}{n}$ - stochastic approximation" [11] procedure which, after the n^{th} round, adjusts by the fraction $\frac{1}{n}$ of the observed miss distance of the n^{th} round. The good asymptotic properties of such stochastic approximation procedures are not surprising when viewed in this light. Similarly, the distribution of the $n+1^{\text{st}}$ impact point X_{n+1}^{n+1}, derived in [10] using multivariate analysis, is now very easily seen to be normal with

mean zero and variance $(1 + \frac{1}{n})\sigma^2 = \frac{n+1}{n}$. That the mean is zero follows from the fact that $\bar{X}_n$ is unbiased for $\mu = E(X^1_{n+1})$. The variance of the aimpoint for the $n+1^{st}$ round is $V(\bar{X}^1_n) = \frac{1}{n}$, and since X^1_{n+1} is independent of $\bar{X}^1_n$, $V(X^{n+1}_{n+1}) = V(X^1_{n+1}) + V(\bar{X}^1_n) = 1 + \frac{1}{n}$.

Remark. It is easily shown, by induction for example, that the n^{th} adjustment in terms of the j^{th} coordinate system (data and shift) is that which places $\bar{X}^j_n$ at zero (i.e. on the target); $\xi^j_n = -\frac{1}{n}\sum_{i=1}^{n} X^j_i$. The earlier observation that $\xi^n_n = -\frac{1}{n} X^n_n$ also follows from this, since $\sum_{i=1}^{n-1} X^n_i \equiv 0$.

3. Strong Optimality of the X-bar Procedure.

We have re-established, by elementary methods, that the X-bar procedure has, under our assumptions of independence, normality and regularity of the target destruction function, the properties

(a) among linear adjustments, it maximizes the probability of destroying the target with each round fired, and

(b) it provides unbiased aimpoints (after the initial round is fired) with minimum variance. It easily follows that also it

(c) maximizes the probability of destroying the target within the first k rounds; $k = 1,2,\ldots,$ and

(d) minimizes the expected number of rounds required to destroy the target. The latter may be established using the identity [4]

$$E(N) = \sum_{j=1}^{\infty} P[N \geq n],$$

where N is the (random) number of rounds required to destroy the target. Evans [6] gives an elegant proof of the intuitively obvious fact that maximizing the probabilities of target destruction with each round minimizes the expected number of rounds required to destroy the target.

Since $\bar{X}$ is a robust estimator of μ, with central limit theorem properties when the X_i's are not necessarily independent and identical, it follows that the X-bar adjustment also, in loose terms

(e) is robust (i.e. tends to operate efficiently even when the underlying assumptions are not met), and

(f) tends to give normally distributed aim points.

We wish to show next that the X-bar procedure is optimal in a class of procedures much wider than the class of linear procedures considered heretofore. The approach is to use a result of Wolfowitz [3], concerning minimax estimators, in a decision theoretic setting. We use the notation and terminology of Ferguson [7]. In the preceding section, it was established that the problem of finding an optimal aimpoint after n rounds have been fired is equivalent to finding an optimal estimator for μ, based on a random sample $X_1^1, X_2^1, \ldots, X_n^1$ from a $N(\mu,1)$ population; once that estimator $\hat{\mu}$ is found, the optimal adjustment for the next round is $-\hat{\mu}$, from the initial aimpoint. Clearly, this <u>problem</u> is invariant under the group of translations* (i.e. roughly speaking, if $\hat{\mu}$ is optimal for μ, $\hat{\mu} + c$ is optimal for $\mu + c$). It is, therefore, extremely reasonable to insist that the <u>solutions</u> to the problem should be invariant (i.e. roughly speaking, if $\hat{\mu}(X_1^1, \ldots, X_n^1)$ is optimal for μ, then $\hat{\mu}(X_1^1+c, X_2^1+c, \ldots, X_n^1+c)$ should be optimal for $\mu + c$). We shall, therefore, restrict our attention to estimators which are invariant under translations; this means essentially that we are indifferent as to where the origin of our coordinate system is placed, relative to the target, before firing begins. (It is intuitively unacceptable for an arbitrary assignment of this sort to affect the procedure to be followed in adjusting fire.)

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*For a discussion of invariant problems and solutions, see Ferguson [7], Definitions 4.1.2 and 4.2.1. It should be noted that, while functional equations of the form $f(x+c) = f(x) + c$ determine linear functions in the single variable case, this is certainly not the case for functions of several variables (see Aczél [1]).

Theorem. In the class I of translation in variant procedures, the X-bar procedure is uniformly best, in the sense that, for any $t > 0$,

$$\max_{\hat{\mu}\in I} P[|\hat{\mu}(X_1^1,\ldots,X_n^1) - \mu| < t]$$

$$= P[|\bar{X}_n^1 - \mu| < t].$$

Our proof makes use of results of Wolfowitz, which are stated in convenient form by Ferguson (Theorem 4.6.1) as follows:

Lemma. If $X_1^1,\ldots,X_n^1$ is a sample from a normal distribution with mean μ and known variance [1, say], then $\bar{X}_n^1$ is a best invariant estimate of μ and a minimax estimate of μ, provided that the loss function $L(\mu,\hat{\mu})$ is a nondecreasing function of $|\hat{\mu} - \mu|$ and that $E[L(0,\bar{X}_n^1)|\mu = 0]$ exists and is finite.

To say that $\bar{X}_n$ is a best invariant estimate of μ means that it is as good as any other invariant estimate: for every μ and any $\hat{\mu} \in I$,

$$E_\mu L(\mu,\bar{X}_n^1) \leq E_\mu L(\mu,\hat{\mu}(X_1^1,\ldots,X_n^1)),$$

where "E_μ" denotes expectation with respect to the joint normal distribution in which the X_i^1 have mean μ. To say that $\bar{X}_n^1$ is minimax estimate of μ means that, roughly speaking, it minimizes the maximum loss we could sustain, regardless of "natures choice" of μ.

Now let

$$L(\mu,\hat{\mu}) = \begin{cases} 0 & \text{if } |\mu - \hat{\mu}| < t \\ 1 & \text{otherwise.} \end{cases}$$

Clearly L satisfies the conditions of the lemma, since for any fixed $t > 0$,

i) L is nondecreasing in $|\mu - \hat{\mu}|$, and

ii) $E_0 L(0,\bar{X}_n^1) = P[|\bar{X}_n^1| \geq t | \mu = 0] \leq 1$.

By the lemma, it follows that, for any μ,

$$E_\mu L(\mu,\hat{\mu}) = P[|\hat{\mu} - \mu| \geq t]$$

is maximized over I by taking $\hat{\mu} = \bar{X}_n^1$, which completes the proof.

The theorem may be interpreted, along the lines of the argument in Section 2, as guaranteeing that the X-bar procedure

(g) has properties (a) through (d) in the class of translation invariant procedures.

As mentioned above, it is difficult to imagine that one would ever want to consider adjustment procedures outside the class I. It should be noted that the class I contains the class of linear procedures as a proper subset; indeed, intuitively speaking, the linear procedures form a "small" subset of I. Thus our theorem may be described in broad terms as dropping the "linear" restriction imposed by Grubbs and Nadler and Eilbott.

Our application of Wolfowitz's results can be extended to a larger class of target destruction functions, by taking the loss function to be

$$L(\mu,\hat{\mu}) = -P_\mu[\text{Procedure using } \xi_n^1 = \hat{\mu}(X_1^1,\ldots,X_n) \text{ destroys the target on the } n+1\underline{\text{st}} \text{ round}]$$

$$= -d(\mu - \hat{\mu}(X_1^1,\ldots,X_n^1))$$

The X-bar procedure will be best (in I), provided that the destruction function is a nonincreasing function of $|\hat{\mu} - \mu|$; existence and finiteness of $E_0 d(-\hat{\mu}(X_1^1,\ldots,X_n^1))$ is automatic. (Relationships between $d(x,y)$ and $d(x)$ are discussed below, in Section 5.)

4. Summary and Conclusions.

The foregoing analyses indicate that an optimal (in several senses) adjustment scheme for the grenadier to pursue consists of shifting the aimpoint by an appropriate fraction of the miss location of the preceding grenade burst. It is significant that this procedure does not require numerical evaluation of miss distances, nor does it require computations with, or manipulations of the sequence of impact points observed as firing proceeds. The procedure we have studied is well known and often used in practice--our contribution has been to determine its optimality under simplifying assumptions.

The most stringent of the assumptions made in our analysis is that the grenadier can produce a shift in aim which is precisely some fraction of an observed miss distance. Some insight into the effects of relaxing these assumptions can be gained by considering the analytical expression in our model for the variance of aimpoint after n rounds. It can be argued that the grenadiers inability to exactly produce the desired shifts in aimpoint will have the effect of increasing variance in aimpoint.

If the grenadier makes normally distributed "errors" with mean 0 and variance τ^2, independent of the actual miss distance, the overall situation is as if the ballistic distribution has variance $\sigma^2 + \tau^2 = 1 + \tau^2$, with an errorless grenadier. That is, under the independence assumption, the mathematical model reduces to the previous case with a larger variance. The variance of the aimpoint for the $n + 1^{st}$ round is now $(1+\tau)/n$, as compared to $1/n$ with the errorless grenadier.

A more realistic model for grenadier error is that the variance decreases with miss distance. Such would be the case, for example, if

the conditional distribution of grenadier error, gives a miss distance x, is uniformly distributed over the interval $x \pm px$, where p represents a proportion parameter measuring the grenadier's skill. This situation places a premium on obtaining rounds near the target, as soon as possible. It is anticipated that the X-bar procedure will be shown to be optimal with a variety of grenadier error models of this general type; we are continuing with work along these lines. Simulations with models of this type appear to support a claim that the variance in aimpoint does not go down like 1/n, as it did in the earlier models we considered. Rather, the variance appears to go down somewhat slower with the latter model; perhaps as $n^{-3/4}$ or so.

These variations in the variance in aimpoint under changes in the grenadier error models and parameters should provide a method of assessing the value of multiple shot grenade launchers. This might be accomplished by assessing the benefits of the multiple round capability (vice a single shot capability) in terms of the grenadier error characteristics, which in turn affects the variance in aimpoint. The aimpoint distribution should still be approximately normally distributed, by the central limit theorem. Hence, with a knowledge of the aimpoint variance and ballistic variance, together with a target destruction function, one can compute the probability of destruction with each round, as well as the expected number of rounds required to destroy the target.

5. Appendix.

We have asserted that an optimal two-dimensional procedure is obtained using optimal one-dimensional procedures in each dimension. The purpose of this appendix is to verify that this is the case, under the following assumptions:

(i) The range X and deflection Y components of the impact point of a round fired at aimpoint $(0,0)$ are independent, symmetric unimodal continuous random variables, with densities $f(x)$ and $g(y)$, respectively.

(ii) The target destruction function $d(x,y)$ symmetric in each argument, where $d(x,y)$ denotes the probability that a round impacting at (x,y) destroys a target at $(0,0)$. Equivalently, under these symmetry conditions, $d(x,y)$ may be viewed as the probability that a round impacting at $(0,0)$ destroys a target at (x,y).

(iii) $\{(x,y): d(x,y) > \alpha\}$ is convex, for all α.

The approach we take is to present two "auxillary" destruction functions, related to $d(x,y)$, which are one dimensional and satisfy the conditions of Anderson's inequality. It is shown that, at each stage of the sequential adjustment procedure, optimal adjustment for the "marginal" problems will give the optimal "joint" adjustment. Suppose the target is at $(0,0)$. The probability of destroying the target, given one aims at (μ_1,μ_2) is

$$p(\mu_1,\mu_2) = P[\text{Target destroyed}\,|\,\text{aim at } (\mu_1,\mu_2)] = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d(x,y)f(x+\mu_1)g(y+\mu_2)dxdy$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} d(x-\mu_1,y-\mu_2)f(x)g(y)dxdy,$$

where $f(x+\mu_1)g(y+\mu_2)$ is the bivariate distribution of impact points with aimpoint (μ_1,μ_2). The problem is to show that this is maximized with $\mu_1 = \mu_2 = 0$; Nadler and Eilbott do so [Lemma 2] using Anderson's inequality, which requires that $d(x,y)$ be symmetric and $\{(x,y): d(x,y) > \alpha\}$ be convex for all α. Now

$$p(\mu_1,\mu_2) = \int_{-\infty}^{\infty} \left[\int_{-\infty}^{\infty} d(x-\mu_1,y-\mu_2)f(x)dx \right] g(y)dy$$

is maximized if it is possible to maximize

$$d_2(y-\mu_2) = \int_{-\infty}^{-\infty} d(x-\mu_1,y-\mu_2)f(x)dx$$

over μ_1 for each value of $y - \mu_2$, independent of the value of $y - \mu_2$, and then to maximize

$$\int d_2(y-\mu_2)g(y)dy$$

over μ_2. Let $d_y(x) = d(x,y)$. As a function of x alone, note that $d_y(-x) = d(-x,y) = d(x,y) = d_y(x)$; and $\{x:d_y(x) > \alpha\}$ is convex, for otherwise $\{(x,y):d(x,y) > \alpha\}$ would not be convex. Thus Anderson's inequality implies that, for the one dimensional problem with ballistic distribution $f(x)$ and <u>any</u> of the destruction functions $d_y(x)$, the maximum of d_2 is attained with $\mu_1 = 0$, independent of μ_2. Now apply Anderson's inequality again for the other dimension, with ballistic distribution $g(y)$ and destruction function $d_2(y)$. [Note that

$$d_2(-y) = \int_{-\infty}^{\infty} d(x,-y)f(x)dx$$

$$= \int_{-\infty}^{\infty} d(x,y)f(x)dx = d_2(y),$$

and $\{y:d_2(y) > \alpha\} = \{y: \int_{-\infty}^{\infty} d(x,y)f(x)dx > \alpha\}$ is convex, again because $\{(x,y):d(x,y) > \alpha\}$ is convex for all α.] It then follows that $\int d(y-\mu_2)g(y)dy$ is maximized with $\mu_2 = 0$.

Thus, given any two dimensional problem satisfying the conditions posed above, an optimal solution is obtained following optimal one dimensional aim strategies at each stage. Note that radial symmetry of the bivariate destruction function $d(\cdot,\cdot)$ is not required.

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